Wave Interference and Standing Waves
Questions concerning today’s youtube video?
Reflections

A sinusoidal wave is generated by shaking one end \((x = L)\) of a fixed string so that the following traveling wave is created:

\[ y_1(x,t) = A \cos(kx + \omega t) \]

What is a possible equation for a wave \(y_2(x, t)\) reflected from the other (fixed) end of the string \((x = 0)\)?

A) \(-A \cos(kx - \omega t)\)  
B) \(A \cos(kx + \omega t)\)  
C) \(-A \cos(kx + \omega t)\)  
D) \(A \sin(kx + \omega t)\)  
E) \(-A \sin(kx - \omega t)\)

String fixed at \(x = 0\) \(\Rightarrow\) \(y(0, t) = 0\) for all \(t\)

\(\Rightarrow\) Need rightward-moving wave and \(y_2(0, t) = -y_1(0, t)\)

\(\Rightarrow\) \(y_2(0, t) = -A \cos(\omega t)\)
Standing Wave – part 1

- A string is clamped at both ends and then plucked so that it vibrates in a standing mode between two extreme positions (a) and (c). Let upward motion correspond to positive velocities. When the string is in position (b), the instantaneous velocity of points along the string is...

A. zero everywhere  
B. negative everywhere  
C. positive everywhere  
D. depends on position
Standing Wave – part 2

- A string is clamped at both ends and then plucked so that it vibrates in a standing mode between two extreme positions (a) and (c). Let upward motion correspond to positive velocities. When the string is in position (c), the instantaneous velocity of points along the string is...

A. zero everywhere
B. negative everywhere
C. positive everywhere
D. depends on position
Plucking a string

- A string on a string instrument plays an A (440 Hz) when plucked. If you put your finger down in the middle of the string, and then pluck, you are mostly likely to hear:

A. Note A an octave higher (880 Hz)
B. Note A an octave lower (220 Hz)
C. Same note/tone
D. Another note [different from (A), (B), and (C)]
E. Nothing \( f = 1/T = v/\lambda \)

\( \Rightarrow \) Reducing \( \lambda \) by 2 increases \( f \) by 2
Two Piano Strings

- Two piano strings have the same tension, are made from the same material, and have the same length, but string B is thicker (has larger diameter) than string A. How will the pitch from string B compare to that of string A?

A. Lower than that of string A
B. Higher than that of string A
C. The same as that of string A
D. Not enough information

\[
f_1 = \frac{1}{2L} \sqrt{\frac{F_t}{\mu}}
\]

\(\mu\) is greater for B
\(\Rightarrow\) Lower \(f\)
A driven string oscillating between two posts (fixed boundaries) exhibits a standing wave pattern with three nodes between the posts. If the tension in the string is increased by a factor 4 with all else held constant, including driving frequency, how many nodes will there be between the posts?

1. One

2. Three

3. Five

4. Seven

5. Nine

Since \( \nu = \sqrt{\frac{F}{\mu}} \), increasing \( F \) by 4 increases \( \nu = \lambda f \) by 2,

\[ \rightarrow \lambda \text{ increased by } 2 \rightarrow 2 \text{ antinodes} \rightarrow 1 \text{ node} \]
A Guitar String

A nylon guitar string of mass 6.48 g and length 0.9 m is supposed to resonate at the fundamental frequency of 80 Hz. What tension should be applied to the string?

A. 59 N
B. 83 N
C. 149 N
D. 317 N
E. 550 N

\[ f_1 = \frac{v}{\lambda} = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{F_t}{\mu}} \]

where \( \mu = \frac{m}{L} \)

\[ \rightarrow F_t = \left(2Lf_1\right)^2 \mu = 4f_1^2Lm \]

\[ F_t = 4(80.0 \text{ Hz})^2(0.9 \text{ m})(0.00648 \text{ kg}) = 149 \text{ N} \]
Resonant rod

A metal rod can be excited to resonate in its audible fundamental mode by rubbing it at the right frequency. What is the nature of this normal mode?

A) Transverse

B) Longitudinal

C) Torsional

The vibration is a sound wave (longitudinal) traveling back and forth along the rod
Varying amplitude \[ y(x,t) = A \sin(kx) \sin(\omega t) \]

A string with both ends held fixed is vibrating in its third harmonic. The waves have a speed of 192 m/s and a frequency of 240 Hz. The amplitude of the standing wave at an antinode is 0.400 cm. What is the amplitude at a point on the string 10.0 cm from its left end?

1) 0.141 cm
2) 0.212 cm
3) **0.283 cm**
4) 0.354 cm
5) 0.425 cm

Amplitude at \( x \) is \( A \sin(kx) \)

\[
\rightarrow (0.400 \text{ cm}) \sin[(7.85 \text{ m}^{-1})(0.100 \text{ m})] = 0.283 \text{ cm}
\]
Elapsed time

A string with both ends held fixed is vibrating in its third harmonic. The waves have a speed of 192 m/s and a frequency of 240 Hz. The amplitude of the standing wave at an antinode is 0.400 cm. How much time does it take the string to go from its largest upward displacement at \( x = 10.0 \text{ cm} \) to its largest downward displacement?

1) \( 1.08 \times 10^{-3} \text{ s} \)  
2) \( 1.48 \times 10^{-3} \text{ s} \)  
3) \( 2.08 \times 10^{-3} \text{ s} \)  
4) \( 2.48 \times 10^{-3} \text{ s} \)  
5) \( 3.08 \times 10^{-3} \text{ s} \)

Need half the period:

\[
\frac{1}{T} = \frac{1}{2f} = \frac{1}{2(240 \text{ Hz})} = 2.08 \times 10^{-3} \text{ s}
\]
Accelerate

A string with both ends held fixed is vibrating in its third harmonic. The waves have a speed of $192 \text{ m/s}$ and a frequency of $240 \text{ Hz}$. The amplitude of the standing wave at an antinode is $0.400 \text{ cm}$. What is the maximum transverse acceleration of the string at $x = 10.0 \text{ cm}$?

1) $6.43 \times 10^{-2} \text{ m/s}^2$
2) $6.43 \times 10^{-1} \text{ m/s}^2$
3) $6.43 \times 10^{+1} \text{ m/s}^2$
4) $6.43 \times 10^{+2} \text{ m/s}^2$
5) $6.43 \times 10^{+3} \text{ m/s}^2$

Each point on the string behaves as a S.H.O.:

$\rightarrow$ Maximum acceleration $a_{\text{max}}(x) = \omega^2 A(x)$

$\rightarrow a_{\text{max}}(10.0 \text{ cm}) = [2\pi(240.0 \text{ Hz})]^2(0.283 \text{ cm})$

$= 6.43 \times 10^{3} \text{ m/s}^2$
Changing frequency

\[ \rho_W = 1000 \, \text{kg/m}^3 \quad \text{and} \quad \rho_A = 2700 \, \text{kg/m}^3 \]

When a massive aluminum sculpture is hung from a steel wire, the fundamental frequency for transverse standing waves on the wire is 250.0 Hz. The sculpture (but not the wire) is then completely submerged in water. What is the new fundamental frequency?

1) 198 Hz
2) 224 Hz
3) 250 Hz
4) 279 Hz
5) 302 Hz

Submerged: \( F_t = mg - F_{buoy} = \rho_{Alum} \, Vg - \rho_{Water} \, Vg \)

\[
\sqrt{\frac{F_t}{mg}} = \sqrt{\frac{\rho_{Alum} - \rho_{Water}}{\rho_{Alum}}} = \sqrt{\frac{2700 - 1000}{2700}} = 0.793
\]