High-precision Absolute Distance and Vibration Measurement using Frequency Scanned Interferometry

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In this paper, we report high-precision absolute distance and vibration measurements performed simultaneously with frequency scanned interferometry using a pair of single mode optical fibers. Absolute distance was determined by counting the interference fringes produced while scanning the laser frequency. A high-finesse Fabry-Perot interferometer (F-P) was used to determine frequency changes during scanning. Two multi-distance-measurement analysis techniques were developed to improve distance precision and to extract the amplitude and frequency of vibrations. Under laboratory conditions, a precision of 40 nm was demonstrated for an absolute distance of approximately 0.45 meters using the first analysis technique. The second analysis technique has capability to measure vibration frequencies ranging from 0.1 Hz to 100 Hz with minimal amplitude on few nanometers order without a priori knowledge. © 2004 Optical Society of America

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1. Introduction

The motivation for this project is to design a novel optical system for quasi-real time alignment of tracker detector elements used in High Energy Physics (HEP) experiments. A.F. Fox-Murphy et al. from Oxford University reported their design of a frequency scanned interferometer (FSI) for precise alignment of the ATLAS Inner Detector. Given the demonstrated need for improvements in detector performance, we plan to design an enhanced FSI system to be utilized for the alignment of tracker elements used in the next generation of electron positron Linear Collider (NLC) detectors. Current plans for future detectors require a spatial resolution for signals from a tracker detector, such as a silicon microstrip or silicon drift detector, to be approximately 7-10 µm. To achieve this required spatial resolution, the measurement precision of absolute distance changes of tracker elements in one dimension should be on the order of 1 µm. Simultaneous measurements from hundreds of interferometers will be used to determine the 3-dimensional positions of the tracker elements.

We describe here a demonstration FSI system built in the laboratory for initial feasibility studies. The main goal was to determine the potential accuracy of absolute distance measurements (ADM’s) that could be achieved under laboratory conditions. Secondary goals included estimating the effects of vibrations and studying error sources crucial to the absolute distance accuracy. A significant amount of
research on ADM’s using wavelength scanning interferometers already exists. In one of the most comprehensive publications on this subject, Stone et al. describe in detail a wavelength scanning heterodyne interferometer consisting of a system built around both a reference and a measurement interferometer, the measurement precisions of absolute distance ranging from 0.3 to 5 meters are \( \sim 250 \) nm by averaging distance measurements from 80 independent scans.

Detectors for HEP experiment must usually be operated remotely because of safety reasons such as intensive radiation, high voltage or strong magnetic fields. In addition, precise tracking elements are typically surrounded by other detector components, making access difficult. For practical application of FSI, optical fibers for light delivery and return are therefore necessary.

We constructed a FSI demonstration system by employing a pair of single mode optical fibers of approximately 1 meter length each, one for transporting the laser beam to the beam splitter and retroreflector and another for receiving return beams. A key issue for the optical fiber FSI is that the intensity of the return beams received by the optical fiber is very weak; the geometrical efficiency is \( 6.25 \times 10^{-10} \) for a measurement distance of 0.5 meter. A novelty in our design is the use of a gradient index lens (GRIN lens) to collimate the output beam from the optical fiber.

We believe our work represents a significant enhancement in the field of FSI in that high-precision ADM’s and vibration measurement are performed simultaneously without a priori knowledge using a tunable laser, an isolator, an off-the-shelf F-P, a fiber coupler, two single mode optical fibers, an interferometer, novel fringe analysis and vibration extraction techniques. Two new multi-distance-measurement analysis techniques are presented, to improve precision and to extract the amplitude and frequency of vibrations. Major uncertainties are also estimated in this paper.

2. Principles

The intensity \( I \) of any two-beam interferometer can be expressed as

\[
I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_1 - \phi_2)
\]

where \( I_1 \) and \( I_2 \) are the intensities of the two combined beams, \( \phi_1 \) and \( \phi_2 \) are the phases. Assuming the optical path lengths of the two beams are \( L_1 \) and \( L_2 \), the phase difference in Eq. (1) is \( \Phi = \phi_1 - \phi_2 = 2\pi|L_1 - L_2|/(\nu/c) \), where \( \nu \) is the optical frequency of the laser beam, and \( c \) is the speed of light.

For a fixed path interferometer, as the frequency of the laser is continuously scanned, the optical beams will constructively and destructively interfere, causing “fringes”. The number of fringes \( \Delta N \) is

\[
\Delta N = |L_1 - L_2|/(\Delta \nu/c) = L\Delta \nu/c
\]

where \( L \) is the optical path difference between the two beams, and \( \Delta \nu \) is the scanned frequency range. The optical path difference (OPD for absolute distance between beamsplitter and retroreflector) can be determined by counting interference fringes while scanning the laser frequency.
3. Demonstration System of FSI

A schematic of the FSI system with a pair of optical fibers is shown in Fig. 1. The light source is a New Focus Velocity 6308 tunable laser (665.1 nm < λ < 675.2 nm). A high-finesse (> 200) Thorlabs SA200 F-P is used to measure the frequency range scanned by the laser, the free spectral range (FSR) of two adjacent F-P peaks is 1.5 GHz, which corresponds to 0.002 nm. A Faraday Isolator was employed to reject light reflected back into the lasing cavity. The laser beam was coupled into a single mode optical fiber by using a fiber coupler. Data acquisition is accomplished using a National Instruments DAQ card capable of simultaneously sampling 4 channels at a rate of 5 MS/s/ch with a precision of 12-bits. Omega thermistors with a tolerance of 0.02 K and a precision of 0.01 mK are used to monitor temperature. The apparatus is supported by a damped Newport optical table.

In order to reduce air flow and temperature fluctuations, a transparent plastic box was constructed on top of the optical table. PVC pipes were installed to shield the volume of air surrounding the laser beam. Inside the PVC pipes, the typical standard deviation of 20 temperature measurements was about 0.5 mK. Temperature fluctuations were suppressed by a factor of approximately 100 by employing the plastic box and PVC pipes.

The beam intensity coupled into the return optical fiber is very weak, requiring ultra-sensitive photodetectors for detection. Considering the limited laser beam intensity and the need to split into many beams to serve a set of interferometers, it’s vital to increase the geometrical efficiency. To this end, a collimator is built by placing an optical fiber in a ferrule (1mm diameter) and gluing one end of the optical fiber to a GRIN lens. The GRIN lens is a 0.25 pitch lens with 0.46 numerical aperture, 1 mm diameter and 2.58 mm length which is optimized for a wavelength of 630nm. The density of the outgoing beam from the optical fiber is increased by a factor of approximately 1000 by using GRIN lens. The return beams are received by another optical fiber and amplified by a Si femtowatt photoreceiver with a gain of $2 \times 10^{10} \text{V/A}$.

The interference fringes from femtowatt photoreceiver and the scanning frequency...
peaks from F-P for the optical fiber FSI system recorded simultaneously by DAQ card are shown in the top and bottom plots of Fig. 2, respectively.

![Femtowatt Photoreceiver Output](image1)

![Fabry-Perot Interferometer Peaks](image2)

Fig. 2. Interference fringes from femtowatt photoreceiver (top) and corresponding F-P peaks (bottom).

### 4. Analysis and Results

For a FSI system, drifts and vibrations occurring along the optical path during the scan will be magnified by a factor of $\Omega = \nu / \Delta \nu$, where $\nu$ is the average optical frequency of the laser beam and $\Delta \nu$ is the scanned frequency. For the full scan of our laser, $\Omega \sim 67$. Small vibrations and drift errors that have negligible effects for many optical applications may have significant impacts on a FSI system. A single-frequency vibration may be expressed as $x_{\text{vib}}(t) = a_{\text{vib}} \cos(2\pi f_{\text{vib}} t + \phi_{\text{vib}})$, where $a_{\text{vib}}$, $f_{\text{vib}}$ and $\phi_{\text{vib}}$ are the amplitude, frequency and phase of the vibration respectively. If $t_0$ is the start time of the scan, Eq. (2) can be re-written as

$$\Delta N = L \Delta \nu / c + 2[x_{\text{vib}}(t)\nu(t) - x_{\text{vib}}(t_0)\nu(t_0)]/c$$  (3)
If we approximate $\nu(t) \sim \nu(t_0) = \nu$, the measured optical path difference $L_{\text{meas}}$ may be expressed as

$$L_{\text{meas}} = L_{\text{true}} - 4a_{\text{vib}}\Omega \sin[\pi f_{\text{vib}}(t - t_0)] \times$$

$$\sin[\pi f_{\text{vib}}(t + t_0) + \phi_{\text{vib}}]$$

(4)

where $L_{\text{true}}$ is the true optical path difference without vibration effects. If the path averaged refractive index of ambient air $\bar{n}_g$ is known, the measured distance is $R_{\text{meas}} = L_{\text{meas}}/(2\bar{n}_g)$.

If the measurement window size $(t - t_0)$ is fixed and the window to measure a set of $R_{\text{meas}}$ is sequentially shifted, the effects of the vibration will be evident. We use a set of distance measurements in one scan by successively shifting the measurement window one F-P peak forward each time. The arithmetic average of all measured $R_{\text{meas}}$ values in one scan is taken to be the measured distance of the scan. For a large number of distance measurements $N_{\text{meas}}$, the vibration effects can be greatly suppressed. Similarly, the uncertainties from fringe and frequency determination dominant in our current system can also be reduced with multiple uncorrelated measurements. Averaging multiple measurements in one scan provides similar precision improvement as averaging distance measurements from multiple independent scans, but is faster, more efficient and less susceptible to systematic errors from drift. In this way, we can improve the distance accuracy dramatically if there are no significant drift errors caused by temperature variation. This multi-distance-measurement technique is called 'slip measurement window with fixed size'. However, there is a trade off in that the thermal drift error is increased with the increase of $N_{\text{meas}}$ because of the larger magnification factor $\Omega$ for smaller measurement window size.

The typical measurement residual versus the distance measurement number in one scan using above technique is shown in Fig. 2(a), where the scanning rate was 0.5 nm/s and the sampling rate was 125 kS/s. Measured distances minus their average value for 10 sequential scans are plotted versus number of measurements ($N_{\text{meas}}$) per scan in Fig. 2(b). It can be seen that the distance errors decrease with an increase of $N_{\text{meas}}$. The standard deviation (RMS) of measured distances for 10 sequential scans is 1.1 $\mu$m if there is only one distance measurement per scan, $N_{\text{meas}} = 1$. If $N_{\text{meas}} = 1200$ and the average value of 1200 distance measurements in each scan is considered as the final measured distance of the scan, the RMS of the final measured distances for 10 scans is 41 nm for the distance of 449828.965 $\mu$m, the relative distance measurement precision is 91 ppb.

In order to extract the amplitude and frequency of the vibration, another multi-distance-measurement technique called 'slip measurement window with fixed start point' was presented. In Eq. (3), if $t_0$ is fixed, the measurement window size is enlarged one F-P peak for each shift, an oscillation of a set of measured $R_{\text{meas}}$ values reflects the amplitude and frequency of vibration. This technique is not suitable for distance measurement because there always exists an initial bias term including $t_0$ which cannot be determined accurately in our current system. In order to test this technique, a PZT transducer was employed to produce vibrations of the retroreflector. For instance, the frequencies of controlled vibration source were set to $1.01 \pm 0.01$ Hz and $10 \pm 0.1$ Hz with amplitude of $0.14 \pm 0.02 \mu$m, respectively.
Since the vibration is magnified for FSI, the expected reconstructed vibration amplitude is $10.0 \pm 1.43 \mu m$. The extracted vibration frequencies and amplitudes using this technique are $f_{vib} = 1.016 \pm 0.002 \text{ Hz}$, $A_{vib} = 9.82 \pm 0.06 \mu m$ (Fig. 3c) and $f_{vib} = 10.075 \pm 0.005 \text{ Hz}$, $A_{vib} = 9.75 \pm 0.09 \mu m$ (Fig. 3d), respectively, in good agreement with expectations.

Fig. 3. Distance measurement residual spreads versus $N_{meas}$ a) for one typical scan, b) for 10 sequential scans, with controlled vibrations of 1 Hz (c) and 10 Hz (d).

Based on our current analysis, vibration frequencies ranging from 0.1 to 100 Hz and amplitudes ranging from $\sim 5 \text{ nm}$ to 1 micron order can be extracted precisely. Vibration frequency far below 0.1 Hz can be regarded as drift error which can not be suppressed by the above analysis techniques. A dual-laser FSI system intended to cancel the drift errors is under study in the lab currently (to be described in a subsequent article).
Nanometer vibration measurement by a self-aligned optical feedback vibrometry technique has been reported. The vibrometry technique is able to measure vibration frequencies ranging from 20 Hz to 20 kHz with minimal measurable vibration amplitude of 1 nm. Our second multi-distance-measurement technique demonstrated above has capability to measure vibration frequencies ranging from 0.1 Hz to 100 Hz with minimal amplitude on few nanometers order without a priori knowledge.

5. Error Estimations

Some major error sources are estimated in the following:

1) Error from uncertainties of fringe and scanned frequency determination. The measurement precision of $R$ (the error due to the air refractive index uncertainty is considered separately below) is given by $(\sigma_R/R)^2 = (\sigma_{\Delta N}/\Delta N)^2 + (\sigma_{\Delta \nu}/\Delta \nu)^2$. For a typical scanning rate of 0.5 nm/s with 10 nm scan range, the full scan time is 20 seconds. The total number of samples for one scan is 2.5 MS if the sampling rate is 125 kS/s. There is about a 4~5 sample ambiguity in fringe peak and valley position due to a vanishing slope and the limitation of the 12-bit sampling precision. However, there is a much smaller uncertainty for the F-P peaks because of their sharpness. Thus, the estimated uncertainty is $\sigma_R/R \sim 1.9 \text{ ppm}$. If $N_{\text{meas}} = 1200$, the corresponding $\Omega^* \sim 94$, $\sigma_R/R \sim 1.9 \text{ ppm} \times \Omega^* / \Omega / \sqrt{1200} \sim 77 \text{ ppb}$.

2) Error from vibrations. The detected amplitude and frequency for vibration (without controlled vibration source) are about 0.3 $\mu$m and 3.2 Hz. The corresponding time for $N_{\text{meas}} = 1200$ sequential measurements is 5.3 seconds. A rough estimation of the resulting error gives $\sigma_R/R \sim 11 \text{ ppb}$.

3) Error from thermal drift. The refractive index of air depends on air temperature, humidity and pressure etc (fluctuation of humidity and pressure have negligible effect on distance measurement for the 20-second scan). Temperature fluctuations are well controlled down to about 0.5 mK(RMS) in our laboratory by the plastic box on the optical table and the pipe shielding the volume of air near the laser beam. For a room temperature of 21 $^0\text{C}$, an air temperature change of 1 K will result in a 0.9 ppm change of air refractive index. For a temperature variation of 0.5 mK in the pipe, $N_{\text{meas}} = 1200$, the estimated error will be $\sigma_R/R \sim 0.9 \text{ ppm} / K \times 0.5 \text{ mK} \times \Omega^* \sim 42 \text{ ppb}$.

The total error from the above sources, when added in quadrature, is $\sim 89 \text{ ppb}$, with the major error sources arising from the uncertainty of fringe determination and the thermal drift. The estimated relative error agrees well with measured relative spreads of 91 ppb in real data.

Besides the above error sources, other sources can contribute to systematic bias in the absolute differential distance measurement. The major systematic bias comes from uncertainty of the FSR of the F-P used to determine scanned frequency range precisely, the relative error would be $\sigma_R/R \sim 50 \text{ ppb}$ if the FSR was calibrated by an wavemeter with a precision of 50 ppb. A wavemeter of this precision was not available for the measurements described here. Systematic bias from uncertainties of temperature, air humidity and barometric pressure scales should have negligible effect.
6. Conclusion

An optical fiber FSI system was constructed to make high-precision absolute distance and vibration measurements simultaneously. A novel design of the optical fiber with GRIN lens was presented which improves the geometrical efficiency significantly. Two new multi-distance-measurement analysis techniques were presented to improve distance precision and to extract the amplitude and frequency of vibrations. An accuracy of 40 nm for a distance of approximately 0.45 meters under laboratory conditions was achieved using the first analysis technique. The second analysis technique is able to measure vibration frequencies ranging from 0.1 Hz to 100 Hz with minimal amplitude on few nanometers order. Major error sources were estimated, and the observed errors were found to be in good agreement with expectation.

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