1) \( \ddot{x} = 0 \rightarrow x = V_0 \cos(\alpha) \ t \\
\dot{y} = -mg \rightarrow y = V_0 \sin(\alpha) \ t - \frac{1}{2} g \ t^2 \\
\text{INTERSECTION WITH HILLSIDE WHEN} \\
y(t) = \beta(x(t))^2 \\
\rightarrow V_0 \sin(\alpha) \ t - \frac{1}{2} g \ t^2 = \beta V_0^2 \cos^2(\alpha) \ t^2 \\
\rightarrow \left[ \frac{1}{2} g + \beta V_0^2 \cos^2(\alpha) \right] \ t^2 = V_0 \sin(\alpha) \ t \\
\rightarrow t = 0 \ (\text{trivial}) \ OR \ t = \frac{2 V_0 \sin(\alpha)}{g + 2 \beta V_0^2 \cos^2(\alpha)} \\
\text{b) THE PEAK OF THE TRAJECTORY IS} \\
\text{REACHED WHEN} \ y = 0 \rightarrow t_{\text{peak}} = \frac{V_0 \sin(\alpha)}{g} \\
\text{WE NEED} \ t_{\text{impact}} < t_{\text{peak}} \\
\frac{2 V_0 \sin(\alpha)}{g + 2 \beta V_0^2 \cos^2(\alpha)} < \frac{V_0 \sin(\alpha)}{g} \\
\rightarrow 2 g < g + 2 \beta V_0^2 \cos^2(\alpha) \\
\rightarrow \cos(\alpha) > \sqrt{\frac{g}{2 \beta V_0^2}} \\
\rightarrow \alpha < \cos^{-1}\left(\sqrt{\frac{g}{2 \beta V_0^2}}\right) \\
\text{c) PHYSICAL SOLUTION REQUIRES} \sqrt{\frac{g}{2 \beta V_0^2}} \leq 1 \\
\rightarrow V_0 \geq \frac{g}{\sqrt{2 \beta}} \)
Before the glue fails, the blocks stretch the spring by an amount \( \Delta \ell \) satisfying

\[ K \Delta \ell = 2mg \]

a) Therefore at time \( t = 0 \), the top of the top block is

\[ y(0) = \ell + \Delta \ell = \ell + \frac{2mg}{K} \]

and its velocity is zero:

\[ \dot{y}(0) = 0 \]

For \( t > 0 \), the equation of motion is

\[ m\ddot{y} = mg - K(y-\ell) \]

or

\[ \ddot{y} + \omega_0^2 y = g + \omega_0^2 \ell \quad (\omega_0 = \sqrt{\frac{K}{m}}) \]

The particular solution to this inhomogeneous equation is

\[ y_p = \ell + \frac{g}{\omega_0^2} \]

which we add to the general solution to the homogeneous equation:

\[ y(t > 0) = A \cos(\omega_0 t) + B \sin(\omega_0 t) + \ell + \frac{g}{\omega_0^2} \]

where we require

\[ y(0) = \ell + \frac{2mg}{K} = \ell + 2 \frac{g}{\omega_0^2} \]

\[ \rightarrow A(1) + B(0) + \ell + \frac{g}{\omega_0^2} = \ell + 2 \frac{g}{\omega_0^2} \]

\[ \rightarrow A = 2/\omega_0^2 = \frac{mg}{K} \]

and

\[ \dot{y}(0) = 0 \]

\[ \rightarrow \omega_0 A(0) + \omega_0 B(1) = 0 \]

\[ \rightarrow B = 0 \]

Therefore,

\[ y(t > 0) = \ell + \frac{mg}{K} (1 + \cos(\omega_0 t)) \]
b) The total energy dissipated as heat is the change in potential energy of the top block from its position at $t=0$ ($y(0) = l + 2mg/k$) to its position as $t \to \infty$ ($y(t) = l + \frac{mg}{k}$).

$$U_{\text{init}} - U_{\text{final}} = -mg(1 + \frac{2mg}{k}) \left( \frac{mg}{k} + \frac{mg}{k} \right)$$

$$\quad + \frac{1}{2} k \left( \frac{2mg}{k} \right)^2 - \frac{1}{2} \frac{mg^2}{k}$$

$$= \frac{m^2 g^2}{k} \left( -2 + l + \frac{l}{2} \right) = \frac{m^2 g^2}{2k}$$

For a right parallelepiped of sides $a, b, c$, the volume $V = abc$, and the total surface area $A = 2(ab + bc + ac)$. Treat the area as a function of three variables $A = A(a, b, c)$ which are constrained by $g(a, b, c) = V - abc = 0$.

Define

$$A'(a, b, c, \lambda) = A(a, b, c) + \lambda g(a, b, c)$$

$$\frac{2A'}{2a} = 2(b + c) - \lambda abc = 0$$

$$\quad \Rightarrow \lambda = \frac{2}{b} + \frac{2}{c}$$

$$\frac{2A'}{2b} = 2(a + c) - \lambda abc = 0$$

$$\quad \Rightarrow \lambda = \frac{2}{a} + \frac{2}{c}$$

$$\frac{2A'}{2c} = 2(a + b) - \lambda abc = 0$$

$$\quad \Rightarrow \lambda = \frac{2}{a} + \frac{2}{b}$$
Therefore, 

\[ a = b = c \rightarrow \text{cube} \quad (a = \sqrt[3]{V}) \]

Since \( A_{\text{cube}} = 6a^2 = 6\sqrt[3]{V^2} \) is finite
and since \( A \) diverges for any of
\( a, b, c \to \infty \) or any of \( a, b, c \to 0 \)
(given \( V = abc \) constraint), we conclude
that the cube minimizes area.